5-1980

A Comparison Of Some Simple Estimation Procedures For Gross Margin Cumulative Distribution Functions

Rien Geuze
Iowa State University

Robert W. Jolly
Iowa State University

A Comparison Of Some Simple Estimation Procedures For Gross Margin Cumulative Distribution Functions

Abstract
The 1970*8 brought major changes to the risk environment in which . agricultural managers function. Increasing international trade, government regulation, market oriented commodity programs, inflation and high capital costs are some of the factors contributing to increasing risk in agriculture. In order to cope with this changing environment, farmers will need to improve their ability to assess risk and return from competing decisions.

Disciplines
Agribusiness | Growth and Development | Industrial Organization | International Business

A COMPARISON OF SOME SIMPLE ESTIMATION PROCEDURES FOR GROSS MARGIN CUMULATIVE DISTRIBUTION FUNCTIONS

Rien Geuze
Department of Agricultural Economics
Agricultural University of Wageningen, Netherlands

and

Robert W. Jolly
Assistant Professor and
Extension Economist

No. 104

May 1980

1.0 Introduction

The 1970's brought major changes to the risk environment in which agricultural managers function. Increasing international trade, government regulation, market oriented commodity programs, inflation and high capital costs are some of the factors contributing to increasing risk in agriculture. In order to cope with this changing environment, farmers will need to improve their ability to assess risk and return from competing decisions.

Stochastic dominance offers one of the most promising approaches to risk analysis and decision making (Anderson *et al.* 1977). Theoretical advances in ranking distribution functions are occuring (Meyer 1977). However, one of the fundamental difficulties that remains is reliable estimation of cummulative distribution functions (CDF) for relevant economic performance criteria.

In this paper we compare four simple approaches to estimating enterprise gross margin CDF's. All of the procedures would be within reach of a farmer with access to a programmable calculator. [1/] We first delineate the major steps and informational requirements for estimating an enterprise CDF. Next the specific estimation procedures are described. In the last section the methods are compared using an empirical example.

2.0 Estimating an Enterprise CDF

In order to estimate the enterprise CDF for a particular performance factor a decision maker must go through the following steps.

1) Identify the choices or decisions to be made. For example should corn or soybeans be planted? Should the producer feed cattle or sell his corn directly?

1/ Anderson (1976) has looked at similar methods that require a large computer.

2) Specify an appropriate enterprise budget that will facilitate the decision. The budget may be expressed algebraically or in the form of an income statement.

3) Identify the major stochastic variables in the budget.

4) Specify the subjective joint distribution function for this variable set.

5) Using theoreotical or Monte Carlo methods estimate the enterprise CDF.

Steps 1-3 don't pose much of a problem. Steps 4-5, given the current state of the art, are virtually insurmountable. Identifying even a two dimensional probability density function (PDF) is difficult and time consuming (Anderson et. al. 1977). Sampling from a multivariate nonnormal joint distribution is not practical when dependence exists (Kleijnen). Therefore, we are forced to rely on various approximations to incorporate information on risk into the enterprise budget.

The approach that we take involves using the triangular probability function as a means for expressing a manager's conditional probability expectations about stochastic variables in the budget. The parameters of the triangular PDF are the mode (m) and the lower and upper extreme points (a, b). Figure 1 shows a hypothetical triangular distribution. 2/ Note that the distribution can be skewed if desired.

2/ Some of the relevant equations for the triangular distribution are given below:

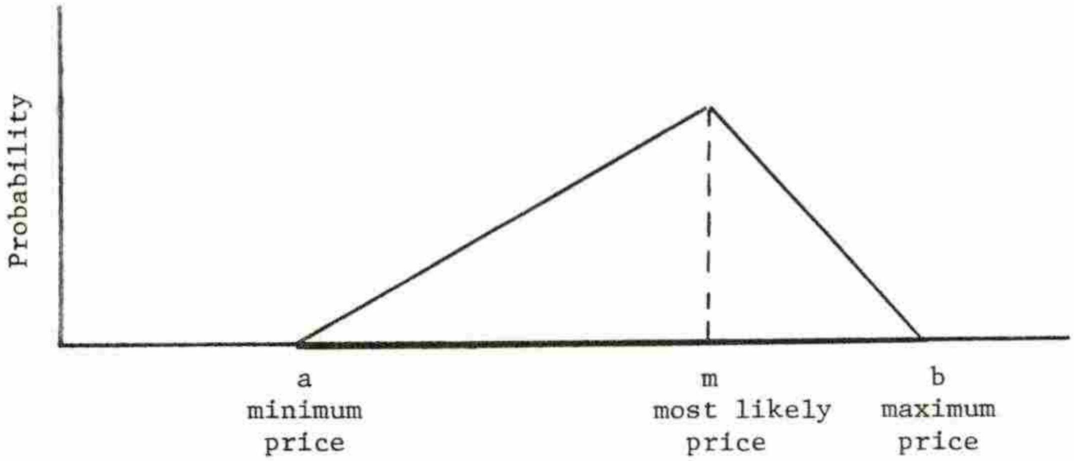
$$\text{CDF:}\quad P\,(x \leq x^*) = \frac{(x^*-a)^2}{(m-a)(b-a)}, \quad a \leq x^* \leq m$$

$$= 1 - \left[\frac{(b-x^*)^2}{(b-m)(b-a)}\right], \quad m \leq x^* \leq b$$

Mean: $\mu = 1/3\ (a + m + b)$

Variance: $\sigma^2 = 1/18\ ((b-a)^2 - (m-a)\ (b-m))$

Figure 1



Identifying and specifying dependence among variables in the budget remains unresolved. However we will examine one possible approach in the following section.

3.0 Estimation Procedures

3.1 Gross margin equation.

We specify the gross margin equation for a particular enterprise as:

$$y = P_o q_o - P_1 q_1 - c \qquad (1)$$

Where:

y = gross margin

P_o = output price

q_o = quantity produced

P_1 = input price

q_1 = quantity used

c = other variable costs

Conceptually any of the arguments of (1) may be random and statistically dependent. We restrict attention to the case where P_o and P_1 are random and may or may not be correlated. We assume the manager can specify marginal triangular distribution functions for P_o and P_1. The quantity variables q_0, q_1 and other variable costs c are assumed to be known and non-random.

3.2 Asymptotically Normal, Without Covariance

This approach relies on the central limit theorum (CLT). The mean of the gross margin is:

$$E(y) = E(P_o)q_o - E(P_1)q_1 - c \tag{2}$$
$$= \mu_y$$

and the variance is simply

$$V(y) = q_o^2 \; V(P_o) + q_1^2 \; V(P_1) \tag{3}$$
$$= \sigma_{y1}^2$$

Although P_o and P_1 are assumed independent they follow unique triangular distributions. The CLT gives some justification for treating the gross margin as a normal variate with mean = μ_y and variance σ_{y1}^2. With this assumption the CDF for the gross margin may be estimated using standard normal tables or a programmable calculator routine (Texas Instruments).

3.3 Asymptotically Normal with Covariance

This method uses the same rationale except the prices are assumed to be correlated. Since managers usually have difficulty specifying subjective covariances we assume a correlation coefficient (ρ) can be estimated from time series data. This path is open to criticism because most agricultural time series are nonstationary and filtering methods are too complex for practical decisions. We will use a short time series to reduce this problem.

In this case the estimate of the mean remains the same as (3). However, the variance is computed as:

$$V(y) = q_o^2 \; V(P_o) + q_1^2 \; V(P_1) - 2q_oq_1 \; Cov(P_o,P_1) \tag{4}$$
$$= \sigma_{y2}^2$$

With a historical estimate of $Cov(P_o,P_1)$, the gross margin is assumed to be distributed normally with mean μ_y and variance σ^2_{y2}. The CDF for y can be estimated as in the preceeding section.[3/]

3.4 Monte Carlo Estimation Without Covariance

This method uses a random number generator routine to draw a sample from the subjective distributions for P_o and P_1. The CDF is estimated from a sample using the sparse data rule (Anderson et. al.). The Monte Carlo simulation of the enterprise budget is performed using a programmable calculator program developed by Geuze.

3.5 Asymptotically Triangular

This approach assumes the distribution of gross margins can be approximated by a triangular PDF. Using the budget and the distributions for P_o and P_1, the manager estimates the most likely gross margin as well as the highest and lowest values. These parameter estimates are used to specify the triangular CDF for gross margin.

3/ Equations for the mean and variance when P_o, P_1, x_o and x_1 are random can be easily determined. With independence we have:

$$E(y) = E(q_o)\ E(P_o) - E(q_1)\ E(P_1) - c$$

$$V(y) = \left[E(q_o)\right]^2 V(P_o) + \left[E(P_o)\right]^2 V(q_o) + V(P_o)\ V(q_o) - \left[\left[E(q_1)\right]^2 V(P_1) + \left[E(P_1)\right]^2 V(q_1) + V(P_1)\ V(q_1)\right]$$

If q_o and P_o are dependent, e.g., in case of yield and price of potatoes and all other correlations are zero then:

$$E(y) = E(q_o)\ E(P_o) + cov\ (q_o, P_o) - E(q_1)\ E(P_1) - c$$

$$V(y) = \left[E(q_o)\right]^2 V(P_o) + \left[E(P_o)\right]^2 V(q_o) + V(P_o)\ V(q_o) + \rho^2 \left\{ \rho\ V(q_o)\ V(P_o) + 2\ E(q_o)\ E(x_o) \left[V(q_o)\ V(P_o)\right]^{1/2} \right\} - \left\{\left[E(q_1)\right]^2 V(P_1) + \left[E(P_1)\right]^2 V(q_1) + V(P_1)\ V(q_1)\right\}$$

4.0 Comparison of the Methods

Using the four procedures we attempted to estimate the gross margin CDF for a beef cattle feeding enterprise. The gross margin was specified using (1) where

y = gross margin, \$/head

P_o = finished cattle price, \$/cwt

q_o = sale weight of steer, cwt

P_1 = price of corn, \$/bu

q_1 = quantity of corn fed, bu/head

c = all other variable costs, \$/head (assumed to be = \$77.14)

Using the estimates for minimum, most likely and maximum prices for beef and corn and using the equation in footnote 2 we obtain the following table.

Prices	Minimum	Most Likely	Maximum	μ	σ^2	σ
P_o	63	67	72	67.333	3.388	1.841
P_1	2.00	2.30	2.80	2.366	.027	.165

Using Iowa on-farm prices of corn and beef cattle for the period 1971-1978 we estimated a correlation coefficient for beef and corn prices $\rho_{P_o, P_1} = .368$. From this we derive $\text{cov}(P_o, P_1) = .368 \times 1.841 \times .165 = .112$.

The estimate of mean gross margin is:

$$E(y) = 11 \times 67.333 + (-64) \times 2.366 - 77.14 = 512$$

The variance without correlation is:

$$V(y) = (11)^2 \times 3.388 + 64^2 \times .027 = 521$$

Accounting for dependence in the estimate gives:

$$V(y) = (11)^2 \times 3.388 + (-64)^2 \times .027 + 2 \times 11 \times (-64) \times .112 =$$

$$= 521 - 158 = 363$$

Note that positive covariance between input and output prices reduces the variance of the gross margin.

The normal CDF's for the above situations are plotted in figure 2.

The Monte Carlo method was used to develop a sample of 49 observations. This permitted an estimate of 2% fractiles using the sparse data rule. Doubling this figure would have added an additional 1% on the probability estimate in the tails.

The triangular approximation approach requires estimating the mode and extreme points of the gross margin. This can be done by selecting the appropriate parameters of the P_o and P_1 distributions. Information on the budgets is given in the following table.

	Quantities		Prices		Gross Margin
	q_o	q_1	P_o	P_1	
Most pessimistic	11	64	63	2.80	437
Most likely	11	64	67	2.30	506
Most optimistic	11	64	72	2.00	587

The CDF was estimated using a programmable calculator program developed by Jolly.

Figure 2 shows a high degree of correlation between the Monte Carlo and independent normal CDF's. This is to be expected from the CLT. When correlation between corn and beef prices is included the reduction in variance does alter the CDF somewhat.

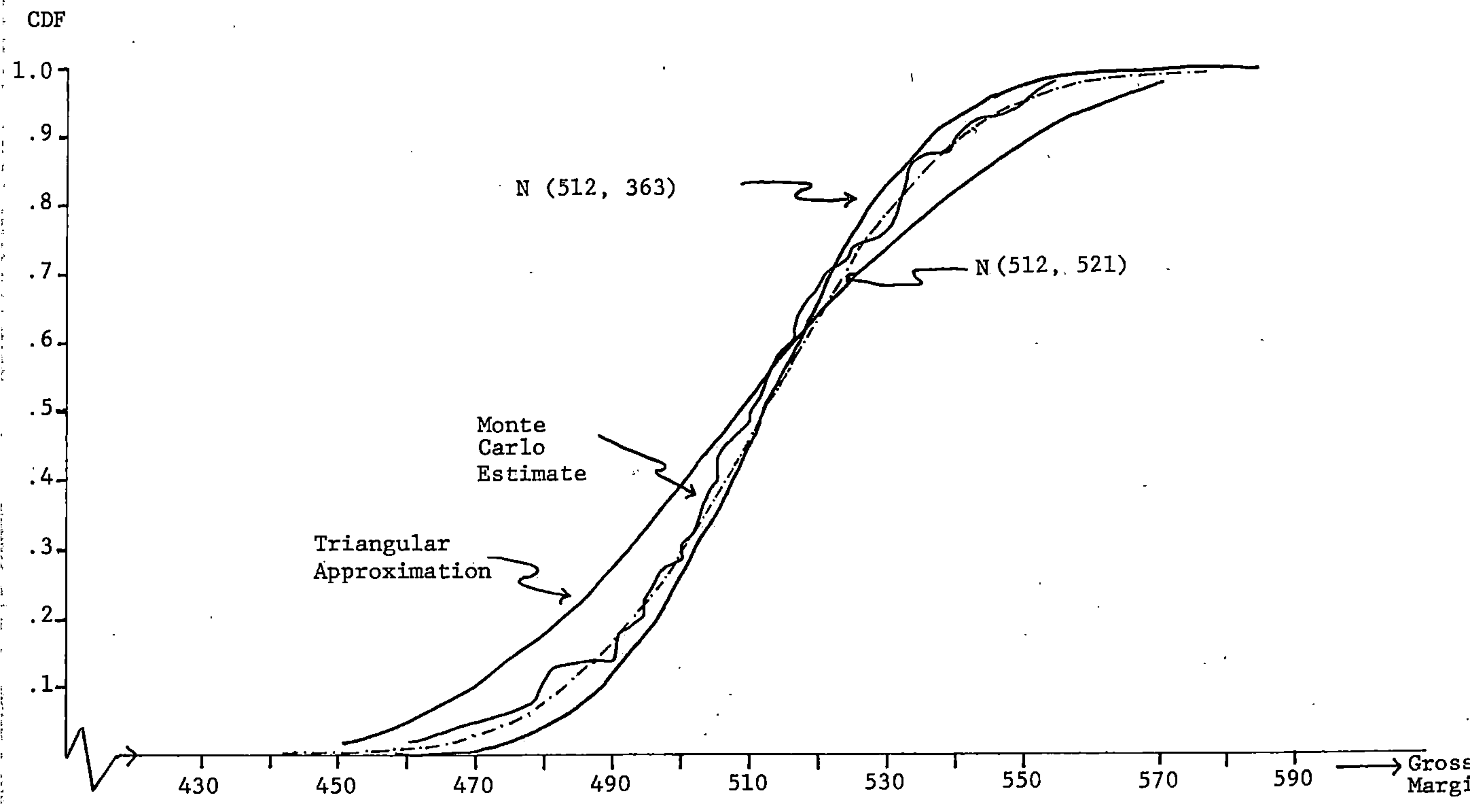
The greatest discrepancy occurs between the triangular method and the other three. The estimated variance of gross margins is much larger, 940 as compared with 363 and 521. The increased probability content of the lower tail could have an impact on an individual's decision. For example suppose a cattle feeder requires a gross margin of $490 to cover feeder and financing costs. The first three methods estimate about a 15% chance of failure. The triangular method estimates a 25% chance of failure. If this risk level is too high, the cattle feeder may decide to sell cash grain and leave the feedlot empty. On the other extreme, the triangular method appears to understate the risk when the probability of failure is high. The triangular approach is the most conservative because it tends to overstate enterprise risk. However being conservative doesn't guarantee risk efficient decisions.

5.0 Further Work

Improving risk assessment skills of agricultural managers requires simple "desk top" analytical tools. We have examined the relative performance of four simple methods for estimating gross margin CDF's. Additional work is called for particularly in accounting for subjective estimates of covariance. The most promising method from an operational perspective involves improvement of Monte Carlo techniques and sampling procedures. The advent of the microcomputer will certainly facilitate these methods. Until estimation methods are improved stochastic dominance will remain a promising concept with little empirical content.

Figure 2

MONTE CARLO TRIANGULAR PROBABILITY DISTRIBUTION FUNCTION FOR GROSS MARGIN ANALYSIS



References

Anderson, J. R. 1976. Methods and Programs for Analysis of Risky Gross Margins. Misc. Publication #5, Dept. of Ag. Economics and Business Management, University of New England, Armidale, Australia.

Anderson, J. R., J. L. Dillon and J. B. Hardaker. 1976. Agricultural Decision Analysis, Iowa State University Press, Ames.

Anderson, Jock R. and Doran, Howard E. 1978. Some properties of simple functions of random variables. Review of Marketing and Agricultural Economics, Vol. 46, (April) 38-47.

Cassidy, P. A., Rodgers, J. L., McCarthy, W. O. 1970. A simulation approach to risk assessment in investment analysis. Review of Marketing and Agricultural Economics, 3-24.

Geuze, Rien. 1980. Monte Carlo risk analysis, FM-1730 (40) - TI. Coop. Ext. Serv., Iowa State University, Ames.

Hertz, David B. 1964. Risk analysis in capital investment. Harvard Business Review, (January) 95-106.

Jolly, Robert. 1980 Triangular probability distribution. FM-1730 (39) - TI. Coop. Ext. Serv., Iowa State University.

Kleijnen, Jack P. C. 1974. A note on sampling two correlated variables. Simulation, Vol. 22 (February) 45-46.

Meyer, Jack. 1977. "Choice among distributions," J. Econ. Theory. 326-336.

Sprow, Frank B. 1967. Evaluation of research expenditures using triangular distribution functions and monte carlo methods. Journal of Industrial and Engineering Chemistry, Vol 59, (July) 35-38.

Texas Instruments. 1977. Applied Statistics, Texas Instruments Inc., Dallas Texas.